GLUE LIKE ROLEOFTHE Λ-PARTICLEIN α-x-Λ CLUSTER STRUCTURE

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Abstract

Energy levels of the lambda- hypernuclei$_{\Lambda}^{6}$He, $_{\Lambda}^{7-8}$Li, $_{\Lambda}^{9}$Be are predicted on the basis of the α-x-Λ three-body model with x = n, d, tand α, respectively. Rearrangement coupled-channels Gaussian basis treatment is used to solve the three-body system. Interactions between the constituent particles are determined so as to reproduce reasonably the observed low-energy properties of the α-x nuclei (^{5}He, $^{6-7}$Li, ^{8}Be) and the existing data of Λ-binding energies of the x+Λ and α+x+Λ systems ($_{\Lambda}^{3-4}$H, $_{\Lambda}^{5}$He and $_{\Lambda}^{6}$He,$_{\Lambda}^{7-8}$Li, $_{\Lambda}^{9}$Be). To solve resonance states with correct boundary condition we have used the proper treatment of the Complex Rotation method. In our calculation, Pauli forbidden states between x and α-clusters were excluded from the solution of Schrödinger equation by implementing the Orthogonality Condition Model (OCM). Structure change of the α+x ordinary nuclei due to the participation of the Λ particle is found to be substantially large.

Keywords: Rearrangement coupled channel method, Complex rotation method, Orthogonality condition model.

Introduction

In nuclear physics, as in almost all branches of physics, the description of resonance is one of the most important tasks. A resonance can be viewed and approached from two different angles; as a delay connected with an enhanced phased shift in a scattering process or as a long-lived but decaying state of a compound system. The main observable characteristics of a resonance are the position and the level width. In the first picture they are to be determined from the phase shift (or cross section) as a function of energy. In the second picture the long-lived state is regarded as an extension of the concept of a bound state in that it is a solution to the Schrödinger equation with purely outgoing asymptotic belonging to complex energy. In our model, it is possible to determine the α-x and Λ-x interactions so as to reproduce all the existing binding of systems in an α-x-Λ three-body system.

Interactions

Interaction between Alpha and x Clusters

We have employed the α-n(t) potentials, parity dependent form with the central and spin-orbit terms, which is introduced by Kanada (Kanada H. et al., (1979)) and Furutani (Furutani H. et al., (1980)). The potential strengths and the range parameters are expressed in the following Gaussian form,

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$$V_{\alpha x}(r)=\sum_{i}^{i_{max}} V_i e^{-\beta_i r^2}+\sum_{i}^{i'_{max}}(-)^{\ell} V_i^{p} e^{-\beta_i^{p} r^2}+\left[\sum_{i}^{i''_{max}} V_i^{\ell.s} e^{-\gamma_i r^2}+i\sum_{i}^{i'''_{max}}(-)^{\ell} V_i^{\ell.s.p} e^{-\gamma_i^{p} r^2}\right]\vec{\ell}.\vec{s}_x \quad (1)$$ where, $\vec{s}_x$ is the spin of neutron(n) or triton(t).The parameters in Eq.(1) are listed in Table (1) and Table (2). The relative angular momentum between α and neutron (triton) is 1 for ground state. The spin of neutron (triton), $\vec{s}_n$ ($\vec{s}_t$) is 1/2. Therefore, the total angular momenta for ground state and first excited state for ^{5}He(^{7}Li) are $3/2^-$ and $1/2^-$ respectively.

The α-d was also introduced by Furutani (Furutani H. et al., (1980)) as follows;

$$V_{\alpha-d}(r)=-74.42\ e^{-(r/2.236)^2}-8.0\ e^{-(r/1.826)^2}\ \vec{\ell}.\vec{s}_d \text{ for all } \ell. \qquad (2)$$

where ℓ is the relative angular momentum between α andd, and $\vec{s}_d$ is the spin of deuteron. The relative angular momenta between alpha and deuteron (ℓ) are 0 for ground state and 2 for 3^+, 2^+, 1^+ excited states. Since the spin of deuteron ($\vec{s}_d$) is 1, the angular momenta and parity for α-d system are 1^+ for ground state and 3^+, 2^+, 1^+ for three excited states. We have calculated the spin orbit $\vec{\ell}.\vec{s}$ coupling effect as $\vec{\ell}.\vec{s}=\left(j^2-\ell^2-s^2\right)/2$.

The size parameters of interaction are in fm^{-2}and strengths are in MeV.

Table (1) The parameters of α-N interaction.

i	**1**	***2***	**3**
β_i	0.36	*0.90*	-
β_i^p	0.20	*0.53*	2.5
γ_i	0.396	*0.52*	2.2
γ_i^p	0.396	*2.2*	-
V_i	-96.3	*77.0*	-
$V_i^{\ p}$	34.0	-85.0	51.0
V_i^{ls}	-20.0	-16.8	20.0
$V_i^{ls,p}$	6.0	-6.0	-

Table(2) The parameters of α-t interaction.

i	**1**	***2***	**3**
β_i	0.0913	*0.1644*	0.2009
β_i^p	0.0913	*0.1644*	0.2009
γ_i	0.28	-	-
γ_i^p	0.28	-	-
V_i	6.9	*-43.35*	-51.7
$V_i^{\ p}$	6.9	43.35	-51.7
V_i^{ls}	-1.2	-	-
$V_i^{ls,p}$	1.2	-	-

The α-α potential is also given in Gaussian form as follows;

$$V_{\alpha-\alpha}(r)=-1.742\ e^{-(r/3.0)^2}-395.9\ e^{-(r/1.898)^2}+299.4\ e^{-(r/1.738)^2}. \qquad (3)$$

These potentials reproduce reasonably well the low-lying states and low-energy scattering phase shifts of the α-x systems.We have also employed the OCM-based cluster model study of light nuclei (Hasegawa A. et al., (1971)).

Lambda- x Potential

In our calculations, we use the effective lambda-neutron Nijmegen model potential of Akaishi (Akaishi Y. (2007)). It is derived from realistic one-boson-exchange YN potential of Nijmegen model D (Nagels. M.M. et al.,(1977)). The original Nijmegen potential is simulated by

Shinmura so as to reproduce the phase shift parameters as the original Nijmegen potential. Then, the effective Y-N potential is derived by the Brueckner Theory. It is parameterized into five-range Gaussian form and parameters for Λ-n singlet even and triplet even state potentials are given in Table (3).

$$V_{\Lambda-n}^{s,t}(r) = \sum_{k=1}^{5} V_k^{s,t} e^{-(\frac{r}{\mu_k})^2} \qquad (4)$$

where s, t stand for singlet and triplet state, respectively.

Table 3 Strength parameters for Λ-n interaction

μ_k (fm)	V_k^s (MeV)	V_k^t(MeV)
0.1800000	47.99645	-46.25826
0.3286335	-272.7777	-43.8389
0.6000000	679.7185	493.1045
1.095445	-160.1574	-136.9770
2.000000	-2.74696	-0.5687829

The phenomenological Λ-d potential was also constructed by adjusting the parameters to fit the value B_Λ=(0.13±0.05) MeV (Bertini R. et al., (1979)).For the Λ-t interaction in ${}_\Lambda^4$H system, we have applied isle type potential which is derived from Dalitz hard core ΛN interaction (Dalitz R.H. et al., (1972)).

Among the various Λ-α potentials, we discuss the potential which is introduced by Myint, Shinmura and Akaishi (Akaishi Y. et al., (2003)) which we will call it MSA Λ-α potential. The required Λ-α potential which based on this effective Λ-N potential is constructed by Hartree-Fock method. It is slightly modified so as to reproduce the experimental binding energy of the ${}_\Lambda^5$He hypernucleus. Lambda-x interactions are expressed in the following Gaussian form;

$$V_{\Lambda-x}(r) = \sum_{k=1}^{3} V_k e^{-(\frac{r}{\mu_k})^2}. \qquad (5)$$

The parameters in Eq.(5) are listed in Table (4).

Table 4 The range parameters of Λ-x interactions are in fm and strengths are in MeV. (x=d,t and α)

System	V_1	V_2	V_3	μ_1	μ_2	μ_3
Λ-d	181.70	-103.40	-30.84	1.08	1.32	1.78
Λ-t	359.2	-324.9	-	1.25	1.41	-
Λ-α	91.0	-95.0	-	1.30	1.70	-

Pauli Suppression Effect in Three-Body System

The Pauli principle between nucleons belonging to α and x clusters is taken into account by the Orthogonality Condition Model; OCM (Saito S., (1969)).The OCM projection operator Vpauli is represented by

$$V_{Pauli} = \lim_{\lambda\to\infty} \lambda \sum_{f} \left|\phi_f(\vec{r})\right\rangle\left\langle\phi_f(\vec{r})\right|. \quad (6)$$

which rules out the amplitude of the Pauli forbidden α-x relative states from $\phi_f(r_{\alpha x})$ the two-body total wave function. The forbidden states are f={0S} for n(p), f={0S, 0P}for d, f = {0S, 1S, 0P, 0D} for t (3He) and f = {0S, 1S, 0D} for α. In the calculation, the strength λ forV_{pauli} is taken to be 10^5 MeV, which is large enough to push up away the unphysical forbidden states into the very high energy region while keeping the physical states unchanged. Harmonic oscillator wave functions are applied for forbidden states.

Complex Coordinate Rotation Method

We use the method of complex coordinate rotation (Gyarmati B. and Kruppa A.T., (1985))to investigate the resonance states. According to the complex rotation method,the following coordinate transformation $r \to r e^{i\theta}$ is carried out, whereθ is a real number called rotation angle.Under this transformation, wave function is defined as

$$\Psi(r) \to \Psi(re^{i\theta}) = \Psi_\theta(r) = \hat{U}(\theta)\Psi(r). \quad (7)$$

Schrödinger Equation under complex rotation is,

$$H_\theta(r)\Psi_\theta(r) = E\ \Psi_\theta(r). \quad (8)$$

Then the asymptotic resonance wave function in Gaussian basis expansion is transformed as

$$\Psi_\theta(r) = \sum_j c_j(\theta) e^{-(r/b_j)^2} \quad (9)$$

Schrödinger Equation becomes

$$H(r)\sum_j c_j(\theta) e^{-(r/b_j e^{i\theta})^2} = E\sum_j c_j(\theta) e^{-(r/b_j e^{i\theta})^2} \quad (10)$$

We have to solve the Schrödinger equation which is the same as bound state system except the range parameter b_j becomes $b_j e^{i\theta}$.

Three-Body Schrödinger Equation

Then Schrödinger equation becomes

$$\left[-\frac{\hbar^2}{2\mu_c}\vec{\nabla}^2_{\vec{r}_c} - \frac{\hbar^2}{2M_c}\vec{\nabla}^2_{\vec{R}_c} + \begin{Bmatrix} V_{23}(r_1) + V_{13}(r_2) + V_{12}(r_3) \\ + V_{Paul} + V_{Coul} \end{Bmatrix}\right]\Psi(\vec{r},\vec{R}) = E\,\Psi(\vec{r},\vec{R}) \quad (11)$$

where c is the channel, c=1, 2 and 3which are described in Fig.1.

The total wave function with angular momentum J and Z component M,$\Psi_{J,M}$ may be expanded in terms of basis functions spanned over the threeRearrangement coupled-channels Gaussian basis treatment (Kamimura M. (1988)) as follows;

$$\Psi_{JM} = \sum_{c=1}^{3} \sum_{i_c I_c l_c L_c} D^{(c)}_{i_c I_c l_c L_c} \phi^{(c)}_{i_c J_c}(r_c)\chi^{(c)}_{l_c L_c}(R_c) \times \left[Y_{l_c}(\hat{r}_c) Y_{L_c}(\hat{R}_c)\right]_{JM} \quad (12)$$

Here l_c (L_c) stands for the angular momentum of the relative motion associated with the

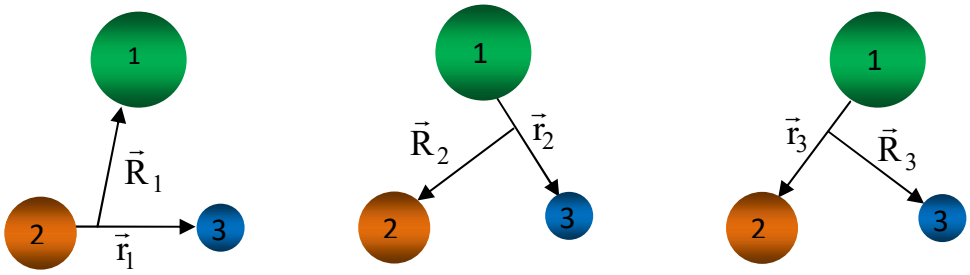
coordinate r_c (R_c), and the bracket$\left[Y_{l_c}(\hat{r_c})Y_{L_c}(\hat{R}_c)\right]_{JM}$ JM represents the vector coupling of the two spherical harmonics.

In Eq. (12),l_c and L_c are restricted as $0 \le \ell_c \le \ell_c^{max}$, $|J - l_c| \le L_c \le J + l_c$, and $(-1)^{l_c+L_c} = (-1)^J$ .The numbers i_c and I_c specify the radial dependences of $\phi^{(c)}_{i_c I_c}(r_c)$ and $\chi^{(c)}_{l_c L_c}(R_c)$, respectively. The form of the radial functions ϕandχ is taken as

$$\phi_{il}(r) = r^{\ell} \exp\{-(r/b_i)^2\}, \qquad b_i = b_1 a^{(i-1)}, (i = 1-n),$$

$$\chi_{IL}(R) = R^{L} \exp\{-(R/B_I)^2,\} \qquad B_I = B_1 A^{(I-1)}, (I = 1-N).$$

These basis functions have been verified to be suited for describing both short-range correlations and long-range tail behaviorsof few-body systems. The eigen energy E and the coefficients D in Eq.(12) are to be determined by the variational method.



Channel (1) Channel (2) Channel (3)

Figure 1 Three rearrangement channels of the α-x-Λ system and their Jacobian coordinate

Relative Density Distribution and Root Mean Square Distance

The relative density distribution $\rho(r_c)$ is obtained by integrating over the other Jacobian coordinate $\vec{R}_c$ and the angular part of $\vec{r}_c$ as follows;

$$\rho(r_c) = \int |\Psi|^2 \, d\vec{R}_c d\hat{r}_c$$

where Ψ is the total wave functionof three-body.

The root mean square distances corresponding to the above density distributions are defined as

$$\tilde{r}_c = \left[\int r_c^2 \, \rho(r_c) r_c^2 \, dr_c\right]^{1/2} \qquad (13)$$

Results and Discussions

Resonance states of x- α cluster

Studies of resonances are indispensable for understanding the unique properties of drip-line nuclei. We have performed the α+n, α+d, α+t and α+α two- body calculation for ^{5}He, $^{6\text{-}7}$Li,

and ^{8}Be. The positions of resonance states in the complex energy plane remains almost unchanged with the variation of rotation angle θ. Pauli operator is applied to push out the forbidden states between α-x clusters. The results and discussion are summarized as follows;

1) The ^{5}He system has been investigated as the typical example of the α-n model. The energies and level widths of these systems we obtained are in good agreement with the experimental values (Tilley D.R. et al., (2002))

2) The excited states of $^{6-7}$Li have been calculated as resonance states of the α-d and α-t system. All our calculated results are in good agreement with the experimental values except for the excited 2^{+} resonance state of ^{6}Li, which lies about 0.735 MeV above the experimental one (2.836±0.022 MeV). Since there are two 2^{+} resonance states with different isospin (I=0 and I=1). Our interaction is independent of isospin and it is not suitable to treat isospin dependent resonance states.

3) The energies and level widths of ^{8}Be have been investigated with α-α model. The obtained binding energies agree with the experimental values within the experimental error (Tilley D.R. et al., (2004)).Therefore to summarize our results, almost all the bound states and excited resonance states we examined are well reproduced by this α-x cluster model with complex rotation method.

Table 5 The energy eigen values of ^{5}He, ^{6}Li, ^{7}Li and ^{8}Be systems

States	Energy Level, E (MeV)		Level Width, Γ(MeV)	
	Our Results	Exp: Results (Tilley D.R. et al., (2002))	Our Results	Exp: Results (Tilley D.R. et al., (2002))
^{5}He($3/2^{-}$)	0.89	0.886±0.008	0.596	0.648±0.006
^{5}He($1/2^{-}$)	2.15	2.068±0.021	5.009	5.570±0.056
^{6}Li(1^{+}) g.s	-1.470	-1.474	0	stable
^{6}Li(3^{+})	0.703	0.712±0.002	0.024	0.024±0.002
^{6}Li(2^{+})	3.571	2.836±0.022	1.514	1.300±0.100
^{6}Li(1^{+})	4.266	4.176±0.050	3.792	1.500±0.200
^{7}Li($3/2^{-}$)	-2.460	-2.467	0	stable
^{7}Li($1/2^{-}$)	-2.065	-1.989±0.003	0.004 eV	0.219±0.006eV
^{7}Li($7/2^{-}$)	2.452	2.185±0.011	0.126	0.069±0.003
^{7}Li($5/2^{-}$)	3.357	4.137±0.207	0.400	0.918±0.046
^{8}Be(0^{+})	0.085	0.091	0.040	0.005±0.25eV
^{8}Be(2^{+})	3.148	3.121±0.010	1.686	1.513±0.002

Total Binding Energies and Lambda Binding Energy ofα-x-Λ cluster

The binding energies of single lambda hypernuclei with α-x-Λ cluster structures have been studied within the coupled-rearrangement channel Gaussian basis treatment. In order to understand the role of Λ particle attached to the ordinary nuclei, it is useful to compare the obtained energy level structures of the α-x-Λ hypernuclei with those of α-x nuclei. The calculated binding energies in two-body and three-body are described in Table (5). Then we can see clearly how the ground and excited states of α-x nuclei are changed due to the participation

of Λ-particle. One sees clearly that injection of Λ-particle leads to stronger binding of the whole system.

Table 6 The total binding energies in two-body and three-body and lambda binding energies; B_Λ

System	B.E (MeV) (α-x)Two-body	B.E (MeV) Three-body	B_Λ(Cal.) (MeV)	B_Λ(Exp.) (MeV)
${}_\Lambda{}^6$He(α-n-Λ)	-0.89	3.29	4.18	4.18±0.10
${}_\Lambda{}^7$Li(α-d-Λ)	1.47	7.31	5.84	5.58±0.03
${}_\Lambda{}^8$Li(α-t-Λ)	2.50	9.32	6.82	6.80±0.03
${}_\Lambda{}^9$Be(α- α -Λ)	-0.09	6.64	6.73	6.71±0.03

Structural change of the α-x nucleus

It is interesting to look at the structural change of the (α-x) ordinary nucleus which occurs due to the participation of Λ-particle. In order to see shrinkage effect in (α-x-Λ) system, we have calculated the two physical properties; the density distribution $\rho(r_{\alpha-x})$ and r.m.s distance between α and x, $\tilde{r}_{\alpha-x}$.Calculated r.m.s distances betweenα and x, $\tilde{r}_{\alpha-x}$ in ordinary nuclei and single Λ-hypernuclei are listed in Table (6). The r.m.s distances of ^{5}He and ^{8}Be ($\tilde{r}_{\alpha-n}$ and $\tilde{r}_{\alpha-\alpha}$) are not calculated since they are resonant states. For ${}^6\text{Li}\rightarrow{}_\Lambda{}^7$Li case, r.m.s distance $\tilde{r}_{\alpha-d}$ changes as 4.10 fm to 3.44 fm. Participation of the Λ-particle gives rise to about 19% reduction of $\tilde{r}_{\alpha-d}$ in three-body system.The r.m.s distance between alpha and triton $\tilde{r}_{\alpha-t}$ reduces from 3.7fm (^{7}Li) to 3.3fm (${}_\Lambda{}^8$Li). It isdue to the glue like role of the attached Λ–particle.

Table 7 Calculated r.m.s distances between α and x(x= n, d, t, α); $\tilde{r}_{\alpha-x}$ (fm) in ordinary nuclei (two-body) and single Λ-hypernuclei (three-body).

System	$\tilde{r}_{\alpha-x}$ (fm) (two-body)	$\tilde{r}_{\alpha-x}$ (fm) (three-body)	Shrinkage (%)
${}_\Lambda{}^6$He(α-n-Λ)	-	5.79	-
${}_\Lambda{}^7$Li(α-d-Λ)	4.10	3.44	19
${}_\Lambda{}^8$Li(α-t-Λ)	3.70	3.30	12
${}_\Lambda{}^9$Be(α- α -Λ)	-	3.78	-

Summary

In this paper, we have investigated the structural change of the α-xordinary nucleus by the participation of the Λ-particle. We have carried out structure calculations of ${}_\Lambda{}^6$He, ${}_\Lambda{}^{7\text{-}8}$Li, ${}_\Lambda{}^9$Be within the frame work of α-x-Λ cluster model (x= n, d, t, α). The three-body calculations of the system were performed by using the Jacobian-coordinate Gaussian basis coupled-rearrangement-channel method. The Pauli forbidden states between α and x particles have been excluded from the solution of Schrödinger equation with OCM model. In our calculation, the optimum set of parameters are b_1= B_1=0.1 fm and a=A=1.996. Calculated lambda binding energies of ${}_\Lambda{}^6$He, ${}_\Lambda{}^{7\text{-}8}$Li, ${}_\Lambda{}^9$Be are in good agreement with the experimental values and injection

of Λ-particle leads to stronger binding of the whole system. Dynamical change of the α-x ordinary nucleus by the participation of the Λ-particle is seen in Λ-hypernucleus; there occurs 19% shrinkage of the α-d and 12% shrinkage of the α-t distance compared with the distance in ordinary nucleus. Therefore we conclude that glue like role of the Λ-particle is important to study the Λ-Hypernuclei.

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References

Akaishi Y.,Private communication.

Bertini R. et al.,(1979). Phys. Lett. **B83** 306.

Dalitz R. H. et al.,(1972).Nucl. Phys. **B47** 109.

Furutani H. et al.,(1980).Prog. Theor. Phys. Suppl. **68** 193.

Gyarmati B. and KruppaA.T., (1985).Phys. Rev. **C34** 1.

Hasegawa A. and NagataS., (1971).Prog. Theor. Phys. **45** 1786.

KanadaH. et al.,(1979).Prog. Theor. Phys. **61** 1327.

Kamimura M.,(1988). Phys. Rev. **A 38**621.

Myint K.S., Shinmura S. and Akaishi Y., (2003).Eur. Phys. J. **A 16** 21.

NagelsM.M., RijkenT.A. and de SwartJ.J.,(1977). Phys. Rev. **D 15** 2547.

SaitoS., (1969).Prog. Theor. Phys. **41**705.

Tilley D.R. et al.,(2002).Nucl. Phys. **A708** 3.

Tilley D.R. et al., (2004).Nucl. Phys. **A745** 155.